

C_n^2 profile from Shack-Hartmann data with CO-SLIDAR data processing

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ABSTRACT

C_n^2 profile monitoring usually makes use of wavefront slope correlations or of scintillation pattern correlations. Wavefront slope correlations provide sensitivity to layers close to the receiving plane. In addition, scintillation correlations allow a better sensitivity to high turbulence layers. Wavefront slope and scintillation correlations are therefore complementary. Slopes and scintillation being recorded simultaneously with a Shack-Hartmann wavefront sensor (SHWFS), we propose here to exploit their correlation to retrieve the C_n^2 profile. The measurement method named COupled SLoDar scIDAR (CO-SLIDAR)¹ uses correlations of SHWFS data from two separated stars. A maximum-likelihood method is developed to estimate precisely the positions and intensities corresponding to each SHWFS spot, which are used as inputs for CO-SLIDAR. First results are presented using SHWFS real data from a binary star.

Keywords: C_n^2 profile; atmospheric turbulence; adaptive optics; wavefront sensing.

1 Introduction

New Adaptive Optics (AO) systems, such as MCAO, GLAO, LTAO, have been conceived to optimize wavefront correction on different fields of view (FOV), but their efficiency very much depends on knowledge of the turbulence vertical distribution. A more precise determination of the turbulence strength profile C_n^2 is therefore needed to improve their performances. A C_n^2 profile can be obtained indirectly from meteorological parameters, but it is more usually measured directly by optical means. These means depend on the number of sources employed and type of data involved. In Generalized SCIDAR (Scintillation Detection and Ranging),² the C_n^2 profile is retrieved from correlation of the scintillation pattern produced by a binary star in a pupil plane. SLODAR (Slope Detection and Ranging)³ uses instead wavefront slope correlations measured on a binary star with a SHWFS.

We propose a new approach for C_n^2 profile measurement named CO-SLIDAR; using a SHWFS, it means both slope and intensity data can be fruitfully utilized. With CO-SLIDAR, slope correlations recorded on two separated stars deliver low-altitude layer sensitivity as a SLODAR. In addition, scintillation correlations and correlations between slopes and scintillation (which is referred further to coupling) deliver high-altitude layer sensitivity. With a limited pupil size and in a single instrument, CO-SLIDAR conjugates the advantages of MASS (Multi-Aperture Scintillation Sensor)⁴ and DIMM (Differentiel Image Motion Monitor),⁵ possibly with better resolution. CO-SLIDAR has been validated numerically.¹ Recently we tested this method with a SHWFS on a single infrared source.⁶ In order to then quantify actual CO-SLIDAR performance on a double star, we here test a new smart estimator that processes subapertures images to extract slopes and intensities.

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In Section 2 we recall the analytical background of a C_n^2 profile measurement based on exploitation of the correlations between SHWFS data: slope correlations, scintillation correlations and their coupling. In Section 3, we present the new estimator based on a maximum-likelihood criterion to measure positions and intensities precisely in a given subaperture. First results using real data from a binary star are presented in Section 4. In Section 5, we sum up our conclusions and open perspectives about CO-SLIDAR.

2 Problem statement with SHWFS slope and scintillation correlations

Given a star with position α in the FOV, a SHWFS delivers a set of wavefront slopes and intensities per frame. The slope computed on the m^{th} subaperture focal image is a bi-dimensional vector $s_m(\alpha)$ with two components $s_m^k(\alpha)$, along the k axis ($k \in \{x, y\}$). Star intensities, denoted $i_m(\alpha)$ and recorded in every sub-aperture m , lead to scintillation index $\delta i_m(\alpha) = \frac{i_m(\alpha) - o_m(\alpha)}{o_m(\alpha)}$ where $o_m(\alpha)$ is the time-averaged star intensity.

For two stars separated from θ , correlations of SHWFS data are empirically estimated from a finite number of frames. Slope correlations $s_m^k s_n^l(\theta)$, scintillation index correlations $\overline{\delta i_m \delta i_n}(\theta)$ and their coupling $\overline{s_m^k \delta i_n}(\theta)$ are stacked in a single dimension covariance vector \mathbf{C}_{mes} . It relates directly to C_n^2 in the problem statement as follows:

$$\mathbf{C}_{mes} = \mathcal{M} \mathbf{C}_n^2 + \mathbf{C}_d + \mathbf{u} \quad (1)$$

where \mathcal{M} is the interaction function; \mathbf{C}_d is the covariance vector of the noises affecting slope and intensity measurements; \mathbf{u} represents uncertainties on \mathbf{C}_{mes} due to the limited number of frames. In Rytov regime, \mathcal{M} is a linear operator M and depends on SHWFS geometry, statistical properties of the turbulence, star separation, distance between sub-apertures and the set of discretized altitudes.⁷ The column vectors of M are formed by the concatenation of weighting functions, for slope correlations, scintillation correlations and their coupling. They can be seen as correlations induced by a turbulent layer at altitude h , for a certain distance between subapertures. Layer contribution to \mathbf{C}_{mes} have been discussed in a recent publication¹ and therefore shall cover that material.

In processing our first real data with CO-SLIDAR (see Section 4), we do not implement \mathbf{C}_d , the covariance vector of noises affecting slope and intensity measurements, that bias the wavefront slope and scintillation correlation estimate. We do have data with a good signal to noise ratio ($SNR \simeq 100$) and the estimator to compute slopes and intensities is noise-free, so \mathbf{C}_d is negligible. Nevertheless, if it were not the case, assuming the system is well calibrated, it would be possible to completely determine \mathbf{C}_d and define a non-biased estimation of the covariance vector, $\hat{\mathbf{C}}_{mes} = \mathbf{C}_{mes} - \mathbf{C}_d$. This would make it possible to rewrite the direct problem:

$$\hat{\mathbf{C}}_{mes} = M \mathbf{C}_n^2 + \mathbf{u} \quad (2)$$

A sampled estimate of C_n^2 , $\tilde{\mathbf{S}}$, is retrieved from the inversion of Eq. 2, assuming that the convergence noise \mathbf{u} is Gaussian. For physical reasons C_n^2 is never negative, so $\tilde{\mathbf{S}}$ minimizes the maximum likelihood criterion J under positivity constraint:

$$J = (\hat{\mathbf{C}}_{mes} - M \tilde{\mathbf{S}})^T C_{conv}^{-1} (\hat{\mathbf{C}}_{mes} - M \tilde{\mathbf{S}}) \quad (3)$$

where $C_{conv} = \langle \mathbf{u} \mathbf{u}^T \rangle$ is the covariance matrix of \mathbf{u} , the convergence noise, due to the limited number of frames.

3 A new estimator for positions and intensities in a SHWFS subaperture

Usually, wavefront slopes are estimated by center of gravity (COG). However, here we also need to measure scintillation indexes for each star in the SHWFS subapertures. We therefore present a new algorithm, called Reconstarfield, that has been developed to estimate positions and intensities corresponding to each SHWFS spot precisely. The principle of the estimator is to minimize a maximum likelihood criterion between the star field image and its model, when the point spread function (PSF) and number of stars is known. The minimized criterion J' is the following:

$$J' \left(\{a_n, x_n, y_n\}_{n=1}^N, b \right) = \sum_{p,q} w(p, q) |i(p, q) - i_0(p, q, \{a_n, x_n, y_n\}_{n=1}^N, b)|^2 \quad (4)$$

where:

- $i_0(p, q, \{a_n, x_n, y_n\}_{n=1}^N, b) = \left[\sum_{n=1}^N a_n h_n(x - x_n, y - y_n) \right]_{\text{III}}(p, q) + b$ is the image model, considering the positions (x_n, y_n) and intensities a_n of the n stars. The index n is included between 1 and N , N being the number of stars in the field. III denotes the delta-comb representing the sampling operation. b is the background level.
- h_n is the PSF of the spot n , taking into account both optical transfer and detector transfer functions.
- $i(p, q) = i_0(p, q) + n(p, q)$ is the recorded image on the detector at pixel (p, q) , namely the sampled image with noise $n(p, q)$, which is assumed to be the sum of photon and detector noises, and can be approximated as Gaussian for the considered fluxes.
- $w(p, q)$ are the weights corresponding to the inverse of the noise variance. They can be inhomogeneous, but here we have supposed $w = 1$, and $w = 0$ can be used to eliminate the influence of bad pixels.

A raw estimation of the star positions, performed by a local COG, starts the minimization. The background level b and spot intensities a_n are estimated analytically for any given spot positions $\{x_n, y_n\}$, which eases and accelerates minimisation of J' . The positions are then determined iteratively for the previous intensities with Levenberg-Marquardt (LM) algorithm.

Reconstarfield has been tested intensively in simulation for binary stars simulated with Gaussian PSF and various separations including close binaries. Results close to reality were obtained with greater precision than with the COG, even in the presence of photon noise and narrow binaries. Realistic SHWFS images were built including photon noise, using a set of 100 images featuring the characteristics of the real data described in Section 4, to compute accuracy on the position evaluation in comparison with that of the COG, and that of intensity retrieval. The precisions are respectively $\sigma_{pos}^2 \simeq 10^{-5} \text{ pixel}^2$ for the positions and $\sigma_{int}^2 \simeq 10^{-5}$ in relative intensity.

4 C_n^2 profile estimation using real data from a binary star

We present here the data processing from a SHWFS installed on the Carlos Sánchez Telescope at the Teide Observatory (Canary Islands). It is a Cassegrain telescope, with a diameter of $D = 1.5 \text{ m}$ and a central obscuration of $0.4D$. The source is a binary star (BS5475), with a separation of $\theta = 5.6''$,

with magnitudes of $V = 4.8$ and 5.9 . SHWFS images consist of 10×10 subapertures, with a diameter of $d = 15\text{ cm}$. The data sequence contains 5000 frames recorded with an iXon EMCCD camera at 330 Hz , so the sequence duration is 15 s . The wavefront is recorded on a $120 \times 120\text{ pixel}$ detector, at a wavelength of $\lambda = 0.6\text{ }\mu\text{m}$. We have assumed monochromaticity of the source and in the following we do not take into account the star spectrum's possible impact on the data. The subaperture field of view contains 12×12 pixels and the image sampling is close to Shannon. We note that SHWFS altitude resolution for a SLODAR, expressed by $\delta h = \frac{d}{\theta}$, is insufficient, since it equals only 5 km and the maximum altitude, $H_{max} \simeq \frac{D}{\theta}$ is too high, about 50 km with such a star separation. We obtain this from the SHWFS geometry with large subapertures of 15 cm in diameter, and from the narrow binary separation. This leads to refined signal processing, presented below, and to optimized SHWFS design and experimentation to be proposed in the future.

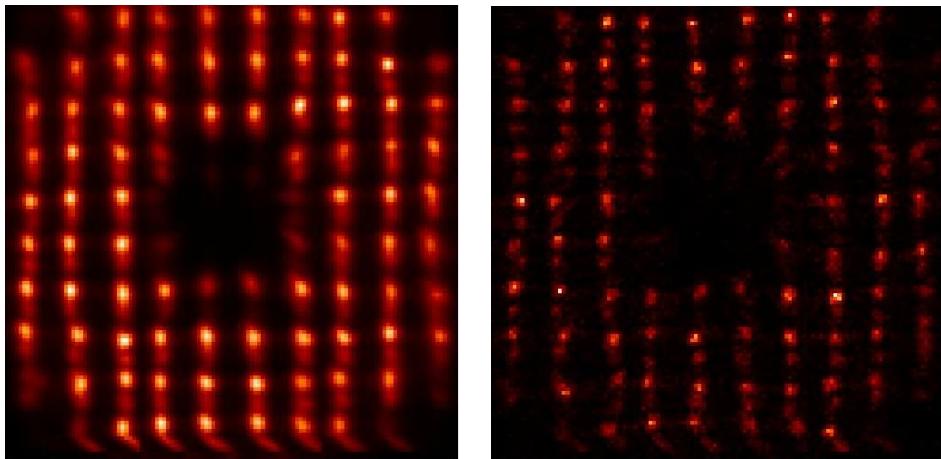


Figure 1. SHWFS long-exposure image (on the left) and short-exposure image (on the right).

Fig. 1 shows the SHWFS long-exposure image and an example of a short-exposure image. As we can see in Fig 1, we cannot use all subapertures, because of the central obscuration and vignetted subapertures at each corner of the images. Only 76 subapertures remain for subaperture image processing. We compute positions (x_n, y_n) and intensities $(i_m(\alpha))$ for each subaperture, for each star, using Reconstarfield. Slopes $(s_m(\alpha))$ are obtained by subtracting the time-averaged position in each direction, for each star. Scintillation indexes are calculated by using $\delta i_m(\alpha) = \frac{i_m(\alpha) - o_m(\alpha)}{o_m(\alpha)}$, where $o_m(\alpha)$ is the time-averaged star intensity. The step of extracting slopes and scintillation indexes being done, we cross-correlate them to feed $\hat{\mathbf{C}}_{mes}$.

C_n^2 profile is retrieved from slope correlations, scintillation correlations and their coupling, by minimizing the J criterion given by Eq. 3. The iterative method is an adaptive step gradient descent. The positivity constraint is implemented by projection and we do not use any regularization function. We first only use the slope correlations to process the data in a "Quasi SLODAR" configuration, then add the scintillation correlations. This data processing is known as Light CO-SLIDAR. Last, we perform a complete CO-SLIDAR analysis, adding the coupling, and what is known as Full CO-SLIDAR. The retrieved C_n^2 profiles corresponding to each approach are presented in Fig. 2 as a function of the altitude.

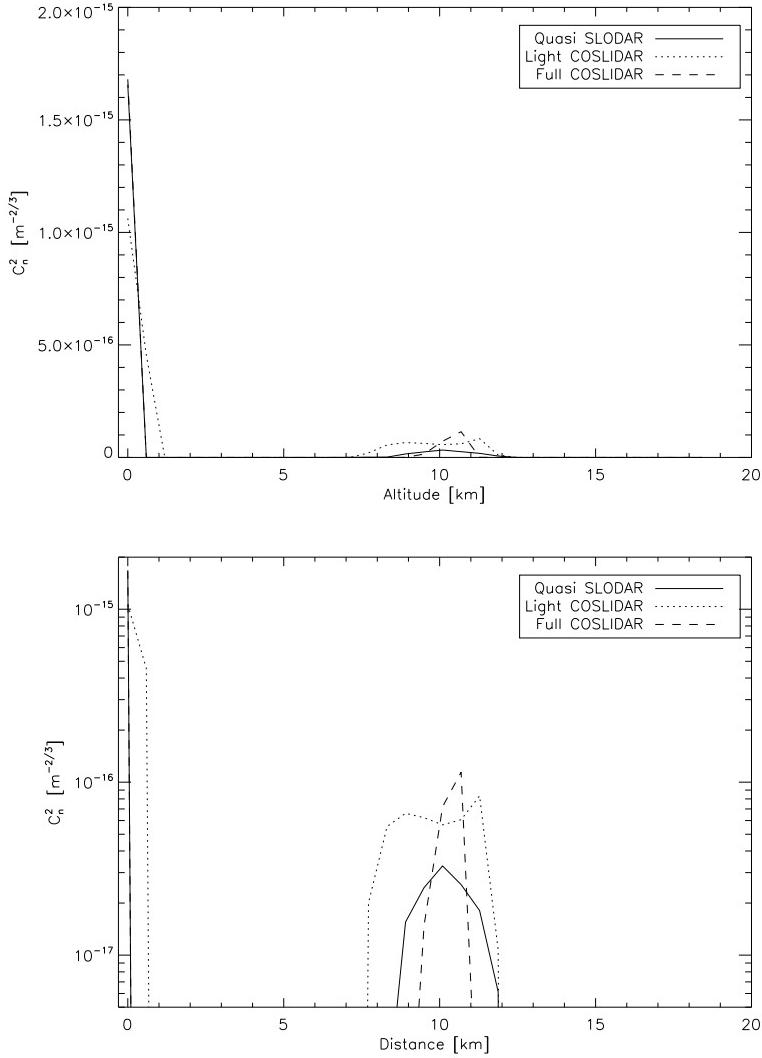


Figure 2. C_n^2 profile results from inversion using CO-SLIDAR data processing. Top: normal scaling on y axis; bottom: log scaling on y axis.

We retrieve a set of 32 values of C_n^2 across 20 km of atmosphere, proceeding with super-resolution. Basically we would have retrieved only 10 values with such a SHWFS geometry. All types of processing detect a strong turbulent layer within the first kilometer. The Quasi SLODAR configuration shows a weak layer at 10 km. With Light CO-SLIDAR, the high-altitude layer is also detected at 10 km and is stronger and larger. The Full CO-SLIDAR method shows a similar layer at the same altitude, but it is thinner. The three methods are in good congruence since they recover layers at the same altitude. Further, this 10–km layer is present if we vary the number of layers reconstructed between 10, 20 and 32, or if we use the scintillation signal only.

Using the Full CO-SLIDAR C_n^2 profile we derive some turbulence parameters. The first one is Fried parameter r_0 defined by:

$$r_0 = \left[0,423 \left(\frac{2\pi}{\lambda} \right)^2 \int_0^{H_{max}} C_n^2(h) dh \right]^{-3/5} \quad (5)$$

We compute r_0 by integrating the restored C_n^2 along the line of sight. Another method is to use the slope variance of the most brilliant star in each subaperture for the whole sequence. Both techniques give $r_0 \simeq 9 \text{ cm}$. The C_n^2 profile also allows to derive the isoplanatic angle θ_0 (Eq. 6) and scintillation rate σ_{scint}^2 (Eq. 7):

$$\theta_0 = \left[2.91 \left(\frac{2\pi}{\lambda} \right)^2 \int_0^{H_{max}} C_n^2(h) h^{5/3} dh \right]^{-3/5} \quad (6)$$

$$\sigma_{scint}^2 = 4 \times 0.56 \left(\frac{2\pi}{\lambda} \right)^{7/6} \int_0^{H_{max}} C_n^2(h) h^{5/6} dh \quad (7)$$

Numerical application of these equations gives $\theta_0 \simeq 2.2''$ and $\sigma_{scint}^2 \simeq 0.09$. The values of r_0 , θ_0 and σ_{scint}^2 are compared with those of a publication presenting synchronized SCIDAR profiles⁸ to the data used in this paper. We get a much smaller r_0 , quite surprising for an astronomical site, and comparable values of θ_0 and σ_{scint}^2 . The C_n^2 profiles involved in computation of turbulence parameters may explain the differences. In any event, the differential scintillation is detectable since the value of σ_{scint}^2 , typical of an astronomical site, is measurable with Reconstarfield and since the binary star separation is larger than the isoplanatic angle θ_0 .

5 Conclusion and perspectives

We have proposed a new approach for C_n^2 profile measurements with a SHWFS, using the information provided by both slope and intensity data. Testing of this concept has begun on experimental data. We have presented here the first results from retrieving the C_n^2 profile with CO-SLIDAR from SHWFS data recorded at the Teide Observatory. A new estimator was employed to compute both positions and intensities in SHWFS subaperture images. The C_n^2 profile is consistent with those of astronomical sites, featuring a strong layer in the first kilometer of altitude and a weak layer at higher altitude (about 10 km). We plan an experience with a better-adapted SHWFS, to retrieve a C_n^2 profile across the first 20 km of atmosphere with a 1 km altitude resolution. This could be done with larger star separation and smaller subapertures.

Additional work should be performed to quantify the number of frames needed to achieve highest accuracy and to take into account detection noise. An *a posteriori* maximum-likelihood criterion should be implemented for the inverse problem, to take a given profile into account (*i.e.* regularization). Outer-scale influence on the accuracy of CO-SLIDAR should be investigated. Determination of wind profile and perhaps outer-scale should also be studied.

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